DEPARTMENT OF CIVIL ENGINEERING

MANUAL
FOR
STRUCTURAL ANALYSIS-II
LABORATORY
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<td>To study the behavior of a portal frame under different end conditions</td>
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<td>4.</td>
<td>To study the behavior of a cantilever beam under symmetrical and unsymmetrical bending</td>
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<td>5.</td>
<td>To determine elastic properties of a beam</td>
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EXPERIMENT: - I

Objective: - To study two hinged arch for the horizontal displacement of the roller end for a given system of loading and to compare the same with those obtained analytically.

Apparatus: - Two Hinged Arch Apparatus, Weight’s, Hanger, Dial Gauge, Scale, Vernier Caliper.

Theory:- The two hinged arch is a statically indeterminate structure of the first degree. The horizontal thrust is the redundant reaction and is obtained by the use of strain energy methods. Two hinged arch is made determinate by treating it as a simply supported curved beam and horizontal thrust as a redundant reaction. The arch spreads out under external load. Horizontal thrust is the redundant reaction is obtained by the use of strain energy method.

\[ H = \frac{5WL}{8r} (a - 2a^3 + a^4) \]

Procedure: -

i) Fix the dial gauge to measure the movement of the roller end of the model and keep the lever out of contact.
ii) Place a load of 0.5kg on the central hanger of the arch to remove any slackness and taking this as the initial position, set the reading on the dial gauge to zero.
iii) Now add 1 kg weights to the hanger and tabulated the horizontal movement of the roller end with increase in the load in steps of 1 kg. Take the reading up to 5 kg load. Dial gauge reading should be noted at the time of unloading also.
iv) Plot a graph between the load and displacement (Theoretical and Experimental) compare. Theoretical values should be computed by using horizontal displacement formula.

v) Now move the lever in contact with 200gm hanger on ratio 4/1 position with a 1kg load on the first hanger. Set the initial reading of the dial gauge to zero.

vi) Place additional 5 kg load on the first hanger without shock and observe the dial gauge reading.

vii) Restore the dial gauge reading to zero by adding loads to the lever hanger, say the load is w kg.

viii) The experimental values of the influence line ordinate at the first hanger position shall be 4w/5.

ix) Repeat the steps 5 to 8 for all other hanger loading positions and tabulate. Plot the influence line ordinates.

**Observation Table:-**

<table>
<thead>
<tr>
<th>Sr.No.</th>
<th>Central load( kg )</th>
<th>0.0</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed horizontal Displacement ( mm )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Calculated horizontal Displacement</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Sample Calculation:** - Central load (kg) =………..
Observed horizontal Displacement (mm). =
Calculated horizontal Displacement = \( H = \frac{5WL}{8r}(a - 2a^3 + a4) \)

\[ = \ldots \ldots \ldots \]

**Precaution :** -

i) Apply the loads without jerk.

ii) Perform the experiment away from vibration and other disturbances.
EXPERIMENT: - 2

- **Objective**: To study the behavior of a portal frame under different end conditions.

**Apparatus**: Portal Frame Apparatus, Weight’s, Hanger, Dial Gauge, Scale, Vernier caliper.

**Formula**: \( P \frac{h}{2} - R_{cy} b = 0, R_{cy} = \frac{ph}{2b} \)
**Theory**: - Structures are categorized as statically determinate or as statically indeterminate. Determinate structures can be analysed additional conditions to solve. The portal frame is an indeterminate structure to several degree of indeterminacy depending on the end conditions.

To know the behavior of any frame it is advisable to know its different deflected shapes under different loading condition, which can be obtained by vertical work energy method analytically.

Consider the portal shown in all the member of which are capable of carrying bending and shear as well as axial force. The legs are hinged at their base and rigidly connected to the cross girder at the top. This structure is statically indeterminate to the first degree; hence, one assumption must be made. Solution of this type of structure based on elastic considerations, show that the total horizontal shear on the portal will be divided almost equally between the two legs; it will therefore be assumed that the horizontal reactions for the two legs are equal to each other and therefore equal to P/22.

The remainder of the analysis can now be carried out by static. The vertical reaction on the right leg can be obtained by taking moment about the hinge at the base of the left leg. The vertical reaction on the left leg can then be found by applying $\Sigma f_y = 0$ to the entire structure. Once the reactions are known, the diagrams of bending moment and shear are easily computed, leading to values for bending moment as given in fig (b). It is well to visualize the deformed shape of the portal under the action of the applied load.

**Procedure**: -

i) Select the end conditions of the portal frame.
ii) Select the point where loading is to be applied (first horizontal than vertical separately).
iii) To obtain the deflected shape of the frame, measure the deflection at various points at legs and the beam separately as detailed below.
iv) Fix the dial gauge and adjust them to zero on one of the leg at various points and measure the vertical distance from the end of these points.
v) Now apply the load at the point selected for loading.
vi) Note down the dial gauge readings
vii) Unload the frame and shift the dial gauge to another leg and repeat the above (4), (5),(6).
viii) Again unload the frame and shift the dial gauge to the beam of the frame and repeated (4), (5), (6).
x) Tabulate the observed reading and sketch the deflected shape for the portal frame on the graph sheet.
x) Repeat the above steps (1) to (9) for various ends conditions and loading conditions to obtain the deflected shape.

**Observation Table:**

<table>
<thead>
<tr>
<th>Points on AC</th>
<th>Distance of Point from C</th>
<th>Dial gauge reading</th>
<th>Deflections (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Initial</td>
<td>Final</td>
</tr>
</tbody>
</table>

Table: - 1

<table>
<thead>
<tr>
<th>Points on AB</th>
<th>Distance of Point from A</th>
<th>Dial gauge reading</th>
<th>Deflections (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Initial</td>
<td>Final</td>
</tr>
</tbody>
</table>

Table: - 2

<table>
<thead>
<tr>
<th>Points on BD</th>
<th>Distance of Point from D</th>
<th>Dial gauge reading</th>
<th>Deflections (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Initial</td>
<td>Final</td>
</tr>
</tbody>
</table>

Table: - 3

**Sample Calculation:**

1) Portal frame with end conditions =
2) Loading point x on AC/AB/BD
   Load applied = kg

**Result** : - The horizontal displacement obtained theoretically and experimentally is nearly same,

**Precaution** : -

i) Apply the load without jerk.
ii) Perform the experiment away from vibration and other disturbances.
EXPERIMENT: - 3

Objective: - To determine the deflection of a pin connected truss analytically & graphically and verify the same experimentally.

Apparatus: - Truss Apparatus, Weight’s, Hanger, Dial Gauge, Scale, Verniar caliper.
Diagram :-

Theory:-The deflection of a node of a truss under a given loading is determined by:
\[ \delta = \Sigma \left( \frac{TUL}{AE} \right) \]

Where, \( \delta \) = deflection at the node point.
\( T \) = Force in any member under the given loading.
\( U \) = Force in any member under a unit load applied at the point where the deflection is required. The unit load acts when the loading on the truss have been removed and acts in the same direction in which the deflection is required.
\( L \) = Length of any member.
\( A \) = Cross sectional area of any member.
\( E \) = Young’s modulus of elasticity of the material of the member.

Here, \( (L/\text{AE}) \) is the property of the member, which is equal to its extension per unit load. It may be determined for each member separately by suspending a load from it and noting the extension.

Procedure: -

i) Detach each spring from the member. Plot extension against load by suspending load from the spring and noting the extension. From the graph, obtain the extension per unit load (stiffness).

ii) For initial position of the truss, load each node with 0.5 kg load to activate each member. Now place the dial gauges in position for measuring the deflections and note down the initial reading in the dial gauges.
iii) Also put additional load of 1kg, at L1, 2kg, L2, and 1kg at L3, and note the final reading in the dial gauges. The difference between the two readings will give the desired deflection at the nodal points. Central deflection y.

iv) Calculate the deflection for the three nodes L1, L2, and L3 from the formula given in Eq. (1) and compare the same with the experimental values obtained in step 3.

v) Draw the Willot – Mohr diagram for deflection and compare the deflection so obtained experimentally and analytically.

**Observation Table:-**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Node Deflection</th>
<th>L1</th>
<th>L2</th>
<th>L3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Initial dial gauge reading ( mm )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>Additional loads ( kgs )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Final dial gauge Reading ( mm )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Deflection (3) – (1) (mm)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Sample Calculation:** - Member = ………….  
L/AE = ………….  
Analytical deflection:= FUL/AE

**Result :-** The theoretical and experimental deflection in various members is found same.

**Precaution :-**

i) Apply the loads without any jerk.

ii) Measure the deflection to the nearest of a millimeter.

iii) Perform the experiment at a location, which is away from any external disturbance.

iv) Ensure that the supports are rigid.
EXPERIMENT: - 4

➤ **OBJECTIVE:**- To study the behavior of a cantilever beam under symmetrical and unsymmetrical bending.

![Diagram of a cantilever beam with labeled axes and angles]

**APPARATUS:**-

Apparatus consist of an angle of size 1” x 1” x 1/8” or in equivalent metric units of length 80cm is tied as a cantilever beam. The beam is fixed at one end such that the rotation of 45° intervals can be given and clamped such that the principal axis of its cross-section may be inclined at any angle with the horizontal and vertical planes. Also arrangement is provided to apply vertical load at the free end of the cantilever and to measure horizontal and vertical deflection of the free end. A dial gauge with magnetic base is supplied with the apparatus.
THEORY:-

A member may be subjected to a bending moment, which acts on a plane inclined to the principal axis (say). This type of bending does not occur in a plane of symmetry of the cross section, it is called unsymmetrical bending. Since the problem related to flexure in general differs from symmetrical bending, it may be termed as skew bending.

One of the basic assumptions in deriving the flexural formula \( f = \frac{MY}{I} \) is that the plane of the load is perpendicular to the neutral axis. Every cross-section has got two mutually perpendicular principal axis of inertia, about one of which the moment of inertia is the maximum and about the other a minimum. It can be shown that a symmetric axis of cross-section is one of the principal axis and one at right angles to the same will be the other principal axis.

For beams having unsymmetrical cross-section such as angle (L) or channel (I) sections, if the plane of loading is not coincident with or parallel to one of the principal axis, the bending is not simple. In that case it is said to be unsymmetrical or non-uniplanar bending.

In the present experiment for a cantilever beam of an angle section, the plane of loading is always kept vertical and the angle iron cantilever beam itself is rotated through angles in steps of 45°.

Considering the position of angle iron wherein the plane of loading makes an angle \( \phi \) with V-V axis of the section, which is one of the principal axes of the section. The components of the vertical load \( P \) along V-V and U-U axis are \( P\cos\phi \) and \( P\sin\phi \) respectively.

The deflection \( \Delta U \) and \( \Delta V \) along U-U and V-V axis respectively are given by

\[
\Delta U = \frac{P \sin \phi \cdot L^3}{3EI_{VV}}
\]

(1)
\[ \Delta V = \frac{P \cos \phi \cdot L^3}{3EI_{uu}} \]  \hspace{1cm} (2)

and the magnitude of resultant deflection \( \Delta \), is given by

\[ \Delta = \sqrt{(\Delta U)^2 + (\Delta V)^2} \]  \hspace{1cm} (3)

and its direction is given by

\[ \beta = \tan^{-1} \frac{\Delta V}{\Delta U} \]

where, \( \beta \) is the inclination of the resultant deflection with the U-U axes. This resultant displacement is perpendicular to the neutral axis n-n but not in the plane of the load P.

\[ \begin{align*}
OO' &= \Delta \\
OP &= \Delta V \\
O'Q &= \Delta X \\
OP &= \Delta U \\
OQ &= \Delta Y \\
\tan \beta &= \frac{\Delta V}{\Delta U} = \frac{O'P}{OP} = \frac{P \cos \phi \cdot L^3}{3EI_{uu}} \\
&= \frac{I_{vv}}{I_{uu}} \cot \phi \hspace{1cm} (5)
\end{align*} \]

For the angle section used in the present experiment \( I_{uu} \) and \( I_{vv} \) can be known from the tables of Bureau of Indian Standards hand book, for properties of standard sections. Therefore for a given angle \( \phi \), the magnitude of angle \( \beta \) can be found out.
The horizontal and vertical components of the deflection can be calculated on the basis of the geometry available

\[ \Delta X = \Delta \cos (\phi + \beta) \]
\[ \Delta Y = \Delta \sin (\phi + \beta) \]  \hspace{1cm} (6)

Similarly,

\[ \Delta X = \Delta U \cos \phi - \Delta V \sin \beta \]
\[ \Delta Y = \Delta U \sin \phi + \Delta V \cos \beta \]  \hspace{1cm} (7)

Therefore, the procedure of calculating the deflections would be

Calculate \( \Delta U \) and \( \Delta V \) using equations (1) and (2).
Compute \( \Delta \) using equations (3).
Compute \( \beta \) using equations (4) and to check the values by using the equation (5)
Calculate the required values of \( \Delta X \) and \( \Delta Y \) using equations (5) and (6) separately.

Procedure:-

Step1:- Clamp the beam at zero position and put a weight of 500gms (5N) on the hanger and take the zero loading on the beam to activate the member.

Step2:- Set the dial gauges to zero reading to measure vertical and horizontal displacement at the free end of the beam.

Step3:- Load the beam in steps of 1kg (10N) up to 4kg and note the vertical and horizontal deflections each time.

Step4:- Repeat the steps (1) to (3) turning the beam through \( 45^0 \) intervals. Problem of unsymmetrical bending will arise only in those cases where the legs of the angle section are in horizontal and vertical positions. In those cases both vertical and horizontal deflections need to measure.
DATA SHEET:

Material of beam = 
Young’s modulus of the material (E) = 
Span of cantilever beam (L) = 
Sectional properties
Size = 
$I_{xx}$ = ......cm$^4$
$I_{yy}$ = ......cm$^4$
$I_{uu}$ = ......cm$^4$
$I_{vv}$ = ......cm$^4$
Area = ......cm$^4$

Data Table

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Angle (degree)</th>
<th>Load (kg)(N)</th>
<th>Observed deflection (mm)</th>
<th>Measured deflections (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>ΔX</td>
<td>ΔY</td>
</tr>
<tr>
<td>1.</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>45º</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>90º</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>135º</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>180º</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>225º</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>270º</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>315º</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

PRECAUTIONS:

• Take care to see that you do not exert force on the free end of the cantilever beam.
• Put the load on the hanger gradually without any jerk.
• Perform the test at a location, which is free from vibration.
EXPERIMENT: - 5

- Objective: To determine elastic properties of a beam.

Apparatus:

The apparatus consists of a flexible beam supported by two rigid supports equally distant from the ends of the beam. This set up provides two over hangs at the ends of the beam.

The beam has notches separated by 0.1 meter distance.

Two hangers are provided at the free ends of the beam to suspend the loads at the ends.

Theory

Where,

- \( a \) = length of overhang on each side
- \( W \) = load applied at the free ends
- \( L \) = main span
- \( E \) = modulus of elasticity of the material of the beam
- \( I \) = moment of inertia of cross section of the beam
Therefore,

\[
EI = \frac{WaL^2}{8y}
\]  
\[\text{------------------ (2)}\]

Also it is known that \(EI\) for the beam = \[
\frac{E \times bd^3}{12}
\]  
\[\text{------------------ (3)}\]

**Procedure:**

1. Find \(b\) and \(d\) of the beam with the help of a vernier calipers
2. Calculate the theoretical value of \((EI)\) from equation (3) and the above parameters.
3. Measure the main span and overhang span of the beam with a scale.
4. By applying equal loads at the free ends of the overhang beam, find the central deflection \(y\). Repeat the process for different values of loads.

**Observations:**

Span of the beam, \(L = \)
Length of overhang on each side, \(a = \)
Width of beam, \(b = \)
Depth of beam, \(d = \)
Modulus of elasticity, \(E = 2 \times 10^6\)

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Equal loads at the two ends (Kg)</th>
<th>Deflection at the mid span of the beam (cm)</th>
<th>((EI)) from Eq. (3)</th>
<th>((EI)) from Eq. (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>